LIFT AND SIDE FORCE ACTING ON A BODY IN A TRANSONIC FLOW

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The lift and side force acting on a body were computed in terms of parameters of a perturbed flow [1] for a dissipative gas, and in [2] for a perfect gas. Three-dimensional flows of a nonviscous gas at sonic velocity at infinity were dealt with in [3]. The viscous laminar wake was considered in [1] where the radial and tangential velocity components of the external flow were correlated with the corresponding velocity components of the wake; the authors of [2] indicate the feasibility of such a correlation.

Below we obtain the distribution of the lift and side force between the wake and the external flow for the cases of a viscous gas and a perfect gas.

In the region of conjugation of solutions, i.e. for the external flow when $r \to 0$, the radial and tangential components of the gas velocity have the form

$$v_r' = -a_* \frac{\varepsilon''}{\Delta''} B_1 \frac{L^2}{r^2} (C_1 \sin\theta + C_2 \cos\theta) \qquad (1)$$

$$v_\theta' = -a_* \frac{\varepsilon''}{\Delta''} B_1 \frac{L^2}{r^2} (C_2 \sin\theta - C_1 \cos\theta)$$

Passing now into the region of conjugation of solutions from the wake, i.e. when $r \to \infty$, we have the corresponding velocity components given by

$$v_r = a_* \varepsilon' \frac{1}{2\pi} \frac{l^2}{r^2} (C_z \sin\theta + C_y \cos\theta) \qquad (2)$$

$$v_\theta = a_* \varepsilon' \frac{1}{2\pi} \frac{l^2}{r^2} (C_y \sin\theta - C_z \cos\theta)$$

Here a_* is the critical velocity of sound, ε'', Δ'' and ε' are numerical parameters considerably smaller than unity, l and L denote certain characteristic dimensions, B_1 is a constant, and C_1, C_2, C_y and C_z are constants associated with the lift and side force. Comparing the velocity components (1) and (2) we obtain the following connection between the constants C_1, C_z and C_2, C_y

$$C_1 = -\frac{\varepsilon'\Delta''}{\varepsilon''} \frac{1}{2\pi B_1} \left(\frac{l}{L}\right)^2 C_z, \qquad C_2 = -\frac{\varepsilon'\Delta''}{\varepsilon''} \frac{1}{2\pi B_1} \left(\frac{l}{L}\right)^2 C_y \qquad (3)$$

We decompose the lift and side force into two terms. One of them represents the part F_i' of forces obtained by integrating the density tensor of the momentum flow π_{ij} over the wake area, and the other represents the part F_y'' obtained by integrating the functions of the external flow. Obviously $F_i = F_i' + F_i''$. According to [1] we have

$$F_y' = -\rho_* a_*^2 \varepsilon' l^2 C_y, \qquad F_z' = -\rho_* a_*^2 \varepsilon' l^2 C_z \qquad (4)$$

for F_i', and using (3) we obtain

$$F_y'' = -{}^1/_2\, \rho_* a_*^2 \varepsilon' l^2 C_y, \qquad F_z'' = -{}^1/_2\, \rho_* a_*^2 \varepsilon' l^2 C_z \qquad (5)$$

for F_i''. Comparing now (4) and (5) we find, that (*)

*) Derivation of the formulas for the total forces F_y and F_z in [1] contained errors.

$$F_y''/F_y' = 1/2, \qquad F_z''/F_z' = 1/2$$

Let us now consider the case when the external flow is described using the framework of the perfect gas conditions. In the transition region, the radial and tangential velocities have (according to [2, 3]) the form

$$v_r'' = -a_* H_{12} r^{-2} (C_y' \cos\theta + C_z' \sin\theta)$$
$$v_\theta'' = -a_* H_{12} r^{-2} (C_y' \sin\theta - C_z' \cos\theta) \qquad (6)$$

where H_{12} is a constant, while C_y' and C_z' are constants associated with the lift and side force. Comparing (2) and (6) we obtain the following relations connecting C_y' with C_y, and C_z' with C_z

$$C_y' = -\frac{\varepsilon' l^2}{2\pi H_{12}} C_y, \qquad C_z' = -\frac{\varepsilon' l^2}{2\pi H_{12}} C_z \qquad (7)$$

Relations (7) together with the contribution of the external flow towards the lift and side force, give

$$F_y'' = -1/2\, \rho_* a_*^2 \varepsilon' l^2 C_y, \qquad F_z'' = -1/2\, \rho_* a_*^2 \varepsilon' l^2 C_z \qquad (8)$$

Formulas (4) and (8) show that the same relation exists for both a perfect and a dissipative gas in the external flow; $2/3 F_y$ and $2/3 F_z$ are determined by the wake, and the external flow determines $1/3 F_y$ and $1/3 F_z$.

The agreement just mentioned is not incidental, since the transverse components of the momentum calculated on the part of the control surface overlapping the wake does not depend on the distance of this surface from the body. The momentum flow carried away by the external flow does not therefore depend on the nature of the gas composing the flow, and is the same for both a perfect and a dissipative gas.

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